Bridging Mechanisms for Reducing Eccentricity on the Real Line

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Abstract

We study a structural modification problem on the real line where n agents are located and a social planner can add a zero-cost bridge to improve connectivity. Our goal is to reduce pairwise distances and individual eccentricities. We introduce bridging mechanisms that place a single bridge and investigate optimal strategies under three objectives: minimizing social cost (sum of all pairwise distances), social eccentricity (sum of individual eccentricities), and maximum cost (worst pairwise distance). For the optimization setting with known locations, we present polynomial-time algorithms that optimally solve each objective. We then move to a mechanism design setting where agents' locations are private and may be misreported strategically. We propose a simple deterministic group-strategyproof mechanism that always connects the two extreme agents. We prove upper bounds on its approximation ratios for all three objectives, including a 2-approximation for maximum cost and $O(n)$ bounds for social cost and social eccentricity. Finally, we establish lower bounds for any deterministic strategyproof mechanism, showing that our guarantees are close to optimal under incentive constraints.

The Model

Consider a set of n agents $N = \{1, 2, \dots, n\}$ on the real line $\mathbb{R}$. Let $\mathbf{x} = (x_1, x_2, \dots, x_n)$ denote the location profile. Without loss of generality, we assume $x_1 \le x_2 \le \cdots \le x_n$. A (deterministic) mechanism $f : \mathbb{R}^n \to \mathbb{R}^2$ maps a location profile $\mathbf{x}$ to a zero-cost bridge (a, b), meaning that there will be no cost for crossing the bridge between a and b. Given a bridge $f(\mathbf{x}) = (a, b)$ with $a < b$, the *cost* from agent i to agent j is their distance under the bridge (a, b)

$$\mathrm{cost}(a, b, x_i, x_j) = \min(|x_i - x_j|, |x_i - a| + |x_j - b|, |x_i - b| + |x_j - a|).$$

An agent's *eccentricity* e_i is the maximum of all the costs between her and all the agents:

$$\mathrm{e}_i(a, b, x_i, \mathbf{x}) = \max_{j \in N} \mathrm{cost}(a, b, x_i, x_j).$$

We want to design strategyproof mechanisms that have good performance guarantees under three objectives: minimizing the social cost, the social eccentricity, and the maximum cost.